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thoroughly studied it will be found to belong to one of these recently differentiated genera. F. H. KNOWLTON.

U. S. National Museum, Washington, D. C.

Mean Values.

MISS PORTER'S kindly criticism (*Science*, June 2) of one point in the article, "Sun-Heat and Orbital Eccentricity" (*Science*, Apr. 28), gives occasion to say a work in regard to mean values. Since the mean value of n quantities is the arithmetic mean of their sum, it would appear at first glance as if the term were a perfectly definite one; but if the quantities to be averaged are successive values of a function of some variable, then clearly their magnitudes depend not only on the nature of the function, but also on the law of variation of the fundamental. Thus, suppose we have the isotherm, $pv = c$, and wish to know the average pressure between the volumes $v = v_1$ and $v = v_2$. It is necessary to make some assumption in regard to the variation of v. If its increments are supposed equal, we understand by the "mean value" of the pressure the average of the pressures corresponding to the values of v. If the volume is assumed to depend in turn on some other variable in such a manner that the abscissa-increments are not equal, the mean value will now be the average of the new series of pressure-ordinates corresponding to the series of values of v arising under the second assumption. Evidently these two means will in general be unequal, but one is just as properly the "real average" as the other. The formula for mean value may be derived by a method even simpler than the usual analytical one as given by Williamson and Todhunter. Let it be required to find the mean value of y where $y = f(x)$ and x is an equicrescent variable. If $y = f(x)$ be treated as a curve referred to rectangular axes, $\int_a^b f(x)\,dx$ is the expression for the area, A, bounded by the X-axis, two ordinates, and the portion of the curve intercepted between the bounding ordinates. Let $A = A'$, where A' is a rectangle whose base equals the base of A. Then the altitude of A' is the average of the ordinates in A. For let

$$\frac{y_1 + y_2 + \dots\dots y_n}{n} = y_0,$$

the average of the series of ordinates.

Then $y_1 + y_2 + \dots\dots = y_0 + y_0 + \dots\dots$ on to n terms.

Multiplying by $\triangle x$ and summing,

$$\Sigma\,(y_1 + y_2 + \dots\dots)\,\triangle x = \Sigma\,(y_0 + y_0 + \dots\dots)\,\triangle x;$$

or, making n indefinitely large,

$$\int_a^b y\,dx = y_0 \int_a^b dx = y_0\,(b - a).$$

But $$\int_a^b y\,dx = A, \text{ hence } y_0\,(b - a) = A',$$

and, since $b - a$ is the base of the rectangle, A', y_0 is it saltitude.

For example, let it be required to find the mean pressure between the volumes v_1 and v_2. If the isotherm is $pv = c$, the area, A, in this case becomes

$$\int_{v_1}^{v_2} \frac{c}{v}\,dv = c \log\left(\frac{v_2}{v_1}\right);$$

its base is $v_2 - v_1$, hence the mean pressure is

$$\frac{c}{v_2 - v_1} \log\left(\frac{v_2}{v_1}\right).$$

This conception of mean values may be readily employed when a curve is expressed in polar coördinates. If $r = f(\theta)$, let x be written for θ and y for r. The Cartesian equation thus arising furnishes a curve which sustains peculiar relations to the original polar curve. The radii-vectores are taken out of their fan-shaped arrangement and placed equi-distant and parallel, with their extremities on the comon line, the X-axis. The pole may be viewed as having developed into this axis, whilst a circle of unit radius with pole as centre has developed into a straight line parallel to the axis, the radii-vectores keeping their normal position with respect to the circle. In finding the mean value of the radius-vector of an ellipse, $d\theta$ being constant, the figure A has three rectilinear sides: $x = 0$, $x = \pi$, and the X-axis. Its fourth side is the curve,

$$y = \frac{a\,(1 - e^2)}{1 + e \cos x}.$$

The base of the figure is π; hence the mean value is

$$\frac{1}{\pi}\int_0^\pi \frac{a\,(1 - e^2)}{1 + e \cos x}\,dx = a\,\sqrt{1 - e^2}.$$

It will be seen that the area-method serves only when the ordinates are equally distributed throughout the area A. In the dynamical problem of the earth's mean distance from the sun it is not θ (or x) which is the equicrescent variable, but t, the time. A must therefore be taken equal to

$$\int_{t_1}^{t_2} r\,dt,$$

for which $r = f(t)$ must be given; but, as is well known, the equation expressing the relation between r and t is transcendental and cannot be written in the form $r = f(t)$. Recourse must therefore be had to other devices for finding the mean distance when the problem is rendered kinematical by taking Kepler's second law into account. ELLEN HAYES.

Wellesley, Mass.

Iron and Aluminium in Bone Black.

WILL you kindly, in your next issue, print the following corrections to my article on "Iron and Aluminium in Bone Black," which has just reached me.

Page 300, first column. In twentieth line (from the bottom of page), after the word "permanent," insert, *and boil*. In nineteenth line (from bottom of page) remove the first two words: "and boil."

In twelfth line (from bottom of page) insert a decimal point between 5 and 0 at end of this line, for the figure must read 5.0 and not 50 grammes.

Page 301, first column. In twentieth line (from bottom of page) transpose after "iron." Instead of "aluminium, or the phosphate" then should stand: or the aluminium phosphate predominates.

J. G. WIECHMANN.

New York, June 7.

Estimated Distance of Phantoms.

IN *Science* of May 19, p. 269, Mr. Bostwick mentions the familiar experiment of binocular combination of regular patterns, such as a tessellated pavement or figured wall-paper, by means of ocular convergence, and states that in his case, although the figures of the phantom thus formed appear smaller, yet contrary to the statements of all other writers they do not appear nearer but farther off than the real object. This seems to me inexplicable if the phantom is really distinct.

As I have very unusual facility in making such binocular combinations, I will very briefly describe an experiment of this kind. I stand now looking down on the tesselated oil-cloth covering the floor of the library. By ocular convergence I slide the two images of the floor over one another in such wise as to combine contiguous figures. After perhaps a brief interval of indistinctness, the pattern appears with perfect clearness at half the distance of the floor and the figures of the pattern of half the real size. The sense of reality is just as perfect as in the case of a real floor at that distance. It seems to me as if I could rap it with my knuckle. Taking now this phantom as a real object, by greater convergence the plane can be brought up higher and higher, until by extreme convergence it is brought within three inches of the root of the nose and seen there with the greatest distinctness in exquisite miniature, the figures being only one-quarter inch in diameter. By relaxing the convergence a little, the phantom-plane may be dropped and caught on lower and lower levels until it falls to its real place on the floor. The combination beyond the plane of the object, and therefore with figures enlarged, is also easy if the figures are small, but never quite so easy as combination on the nearer side.

These phenomena are as easy to me as any ordinary act of sight. No device of any kind, such as use of pencil or finger to fix the point of sight is at all necessary. I can watch the double images approach, combine, pass over, combine with the next figure, etc., with the greatest ease and certainty. Moreover, the sense of reality and of exact distance is as perfect as that of any other object.

In young normal eyes great difficulty is often experienced in getting this perfect perception of distance because the phantom is not perfectly clear. The reason is this: The two adjustments of the eyes, the axial and the focal, are invariably associated in every act of sight. Therefore, in the experiment the eyes are accommodated to the point of ocular convergence, i.e., the distance of the phantom. But the light comes from a greater distance, viz., from a real object—the floor. The retinal image, therefore, is not distinct and the figures are blurred. I no longer, now, suffer from this difficulty, because I have become presbyopic, and have, therefore, lost the power of accommodation. The clearness of the phantom is perfect almost immediately. When I was younger, there was always a considerable interval before the phantom became clear. The clearing up was the result of a dissociation of these two consensual adjustments. While the axial adjustment remained adapted for the distance of the phantom, the focal adjustment (accommodation) was changed to the distance of the real object. Now this dissociation of two adjustments so invariably associated in every act of sight, is difficult for most, and impossible for many persons. But until this dissociation is effected and the phanton becomes perfectly clear, the sense of reality, and especially the perception of distance, will be imperfect and vacillating. The use of glasses adapted to distinct vision at the distance of some one of the possible phantoms will make that particular phantom clear.

Now this clear perception of the distance of a phantom, nearer and smaller in proportion to the degree of ocular convergence, is, of course, not peculiar to me. All writers on the subject record the same result. All my pupils who succeed at all in binocular combinations get the same result. I am sure I have tried hundreds, I might almost say thousands, and always with the same result. This result is, therefore, normal and in complete accord with the laws of vision. For near objects, there are two modes of estimating distance, viz., by axial convergence (binocular perspective) and by accommodation (focal perspective). Now, of these two, the former is by far the more exact, and therefore takes control of judgment of distance if the two are not in accord. This is proved by naked-eye combination of ordinary stereoscopic pictures by ocular convergence. In such cases, we have the phenomenon of inverse perspective. The judgment of relative distance by axial convergence completely reverses the real relative position of objects. Binocular perspective overrides every other form of perspective, whether focal, or mathematical, or aërial, and comes out victorious in spite of the absurdity or even impossibility of its results.[1]

Now, in the case of phantoms, axial convergence fixes the distance. But this fixes also the size; for the apparent size of anything is a product of the retinal image multiplied by the estimated distance. The size of the figures will be small in proportion to the nearness of the phantom. This is in exact accord with the laws of vision. But Mr. Bostwick says, that in his case the figures seem smaller and yet more distant than the real object. He explains this, if I understand him aright, by the fact that in the dissociation of the axial and focal adjustments, while most persons follow the axial, he follows the focal adjustment, in estimating distance. Near objects require greater accommodation; but there is no such accommodation in this case, therefore the objects judged by this test will not seem nearer. But, again, since

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[1] If anyone is specially interested in this subject, he will find it fully treated in my little volume, entitled "Sight," volume 31 of the International Scientific Series.

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they seem smaller, and since distant objects also seem smaller, they will seem more distant.

There are two objections to this explanation. 1. The accommodation is for the distance of the real object, as is proved by the distinctness. Why, then, should the object seem farther? 2. Again, distant objects seem smaller only because their retinal images are smaller; but this is not so in the case under consideration.

In justification of his view, Mr. Bostwick says that "in monocular vision an object appears distant or near according as the eye is fixed respectively on something nearer than it or something beyond it." I am familiar with the fact here referred to, but in this case the appearance of greater or less distance is so imperfect that it can hardly be called estimate. It may seem farther or nearer almost at will. It is a matter of fancy, not a sober certainty of rational judgment. In fact, there is no ground for forming any judgment.

Although Mr. Bostwick speaks of his estimate of the distance of the phantom as "distinct," yet I cannot but think that, for want of complete dissociation of the axial and focal adjustments the image is not quite sharp; and that, if he got the same sharp, realistic image which I get, he would see the distance as I see it. Of course, there is no disputing about how things seem to different observers any more than there is about tastes; but nevertheless, there are some things which are normal and reducible to intelligible law, and some not. Mr. Bostwick's case may be abnormal, but I think probably not. I well know how illusive binocular phenomena are. He will, I am sure, pardon me for thinking that with more practice in experiments of this kind he will come to see things as others see them.

JOSEPH LECONTE.

Berkeley, Cal., May 27.

A Rain of Fishes.

DURING a recent thunder-storm at Winter Park, Fla., a number of fish fell with the rain. They were sunfish from two to four inches long. It is supposed that they were taken up by a waterspout from Lake Virginia, and carried westward by the strong wind that was blowing at the time. The distance from the lake to the place where they fell is about a mile.

THOMAS R. BAKER.

AMONG THE PUBLISHERS.

MACMILLAN & CO. have published a brief biography of the late English anatomist, William Kitchen Parker, written by his son, T. Jeffery Parker. It begins with an account of his birth and early life on his father's farm, and then of his schooling and his apprenticeship, first to an apothecary and afterwards to a surgeon. With his strong inclination for biological studies, it was natural that he should choose medicine as his profession; but it is evident, as indeed his biographer admits, that he had no great love for his profession and only moderate success in the practice of it His prime interests in life, apart from his family, were two things not often found in conjunction at the present day, science and Wesleyan religion; and he seems to have been equally devoted to both and to have found no incongruity between the two. In biology he was largely self-taught; but a few discerning friends saw that he was capable of important original work, and assisted him in the prosecution of such work. He became a member of the Zoölogical Society and afterwards a fellow of the Royal Society; but the position that proved the most useful to him was the Hunterian professorship of anatomy and physiology in the Royal College of Surgeons, because it not only gave him the opportunity to lecture on his favorite subjects, but also added to his otherwise moderate income. His principal scientific work, his researches on the skull, is described at some length in this book, and there are briefer notices of his other studies and a bibliography of all his published works. His principal fault as a scientific writer, his son thinks, was his complicated style; his topics being arranged in a disorderly way and his sentences hastily constructed. Yet biologists will doubtless echo the words of the Royal Society that he was "an unworldly seeker after truth . . . whose beneficent influence will ever be felt in a wide-spreading and advancing science."

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The Rev. A. C. Waghorne, New Harbor, Newfoundland, wishes to sell collections of Newfoundland and Labrador plants, all named by competent botanists. He is going on a missionary journey along the Labrador coast, from the middle of July till October, and in return for much needed aid towards (Episcopal) Church purposes in that region, will be glad to be of service to any botanists who may write to him. Letters posted in the U. S. up to July 1 will reach him at the above address, and if posted later will be forwarded.

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For exchange—I wish to exchange cabinet skins of Californian birds or mammals for any book on the following list, books if second-hand to be in good order. Manual of Vertebrates, fifth edition, D. S. Jordan; Nests and Eggs of North American Birds, Oliver Davie; Marine Mammals of the West Coast of North America, C. M. Scammon; The United States and Mexican Boundary Survey, Vol. II., Zoology, S. F. Baird. F. Stephens, Witch Creek, San Diego Co., Cal.

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